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TEACHERS' USE OF VERBAL LANGUAGE TO EVOKE VISUALIZATIONS IN MULTILINGUAL MATHEMATICS CLASSES

Abstract

Research into the importance of teachers' use of verbal language in multilingual classes where teaching and learning is done in a language other than the learners' first, is crucial. Such research aimed at improving multilingual learners' access to conceptual understanding of mathematical concepts is becoming increasingly urgent. In this paper, we specifically report on a study that inter alia focused on how three purposively selected South African (Eastern Cape) Grade 11 multilingual mathematics teachers used learners' first language, through code switching, to evoke visualizations for promoting conceptual understanding during the teaching of geometry and trigonometry. The study found that conceptual understanding was enhanced when teachers spoke in their learners' first language using mostly everyday familiar words to evoke visualizations of some geometry and trigonometry concepts. Teachers also used isiXhosa terminology, the dominant language of the Eastern Cape Province in South Africa, to describe some mathematical constructs. A number of instances were noted in which teachers used pictorial illustrations from the learners' environment when words or phrases were not immediately available to them, to promote conceptual teaching of mathematics. However, none of the strategies used were planned for; they were used spontaneously and on an ad hoc basis. We concluded that in multilingual mathematics classes, teachers should choose their verbal language carefully and purposefully to precisely demonstrate and make visible the intended mathematical ideas. Such use of verbal language is particularly important in situations where teachers cannot bring the actual or physical artefact of the mathematical idea to class. The underpinning theory that framed this paper was situated-sociocultural theory.

Keywords: *Language, Multilingual, Code switching, Visualization, Mathematics*

1. Introduction

In South Africa, the importance of using verbal language in multilingual mathematics classes to improve learners' access to conceptual understanding has been a subject of research and is becoming increasingly urgent. Teachers' verbal language is crucial, as it is used among other things to evoke relevant mental images necessary for understanding mathematics concepts (Mesarosova & Mesaros, 2011).

During teaching, verbal language is paramount and often indispensable in triggering mental images necessary for constructing meaning when actual representations are not immediately available (Marentette, 2018). Sadoski and Paivio (2013) aptly assert that without the use or activation of mental representations in the teaching and learning of mathematics, no meaning can be achieved. Teachers' choice of words both in the language of learning and teaching (LOLT) and the learners' home language should ideally be used to create meaning, thereby making learning possible. We assert that the teachers' language should activate appropriate mental images and other representations to support and mediate conceptual learning in multilingual classes (Smith, Bill & Raith, 2018). In this study, we define multilinguals as those individuals who participate in multiple-language communities (Moschkovich, 2002) and we specifically looked at teachers' choice of words and visualizations in the learners' home language when teaching mathematics to English second language learners. We adopt Arcavi (2003: 217)'s definition of visualizations as

> *… the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.*

Our interest was to understand how selected secondary school teachers use words and visualizations in the learners' first language to stimulate visual thinking in their learners and to enhance meaning making during teaching. An adoption of "a situated view of the meaning of words…, that words have multiple and ever-changing meanings created for and adapted to specific contexts of use" (Gee, 1999: 40) was made in this study. We were inspired by our own observations and experiences that when teaching, the language used to describe a particular concept may actually result in different and often unanticipated understandings of the same concept. For example, the word 'seven' may suggest a quantity aspect (how many), a spatial aspect (seven units above, seven units below), equivalence, distance (seven units from), time (seven days in a week) combinations and units of measurement, among others.

In situations where teaching is done in a language that is neither the teachers' nor the learners' first language, research has repeatedly encouraged the use of learners' first language as a linguistic resource during teaching (Barwell, 2018) and in the context of this paper, a visual-generating mechanism to illustrate a mathematical idea. In South Africa's many rural and township schools, mathematics teaching occurs predominantly through code switching which Adler (2001) refers to as the alternate use of two or more languages in one conversation. This is because learners in many rural and township schools are LOLT learners (Setati, 2008), hence teachers resort to code switching when teaching multilingual classes. While research has shown that code-switching is crucial and has benefits (Barwell, 2018; Probyn, 2006), the use of learners' first language through code switching needs to help create appropriate meaning and foster conceptual understanding.

The teacher's use of preferably the learners' first language to help them visualize mathematical ideas, is thus important. It facilitates not only connection-making processes to everyday life situations, but also within mathematics, and between different disciplines. It enhances conceptual understanding and meaning making at many levels. There is thus a need for research on how language can be used to specifically enhance visualization in mathematics classes where the LOLT is not the pupils' first language. One of the reasons why borrowing code switching is a prevalent mode of teaching in classrooms where the learners' home language is not the LOLT (Chikiwa & Schäfer, 2016; Mokgwathi & Webb, 2013) is because there is no formal mathematics register in that home language. It is thus prudent to explore how a language whose mathematics register is not formally developed, can be more effectively used to enhance visualization in mathematics pedagogy. Research by Kasmer and Billings (2017) and Kersaint, Thompson and Petkov (2009) has shown that learners whose language is not sufficiently developed to function as the LOLT may benefit positively from teaching and learning that uses other means of communication such as code switching and non-verbal information – i.e. using visualization in this case.

It is against this backdrop that we view teachers' incorporation of learners' first language through code switching as a potential resource available for multilingual teachers and their learners to use to enhance conceptual understanding during the teaching and learning (specifically visualizing) of mathematics. Hence, this study seeks to answer the following questions:

> *How do mathematics teachers use learners' first language through code switching to evoke visualizations to promote conceptual understanding?*
>
> *What verbal illustrations for promoting conceptual understanding in learners' second language do teachers use to enhance visualization of mathematical concepts during teaching?*

2. Literature Review

2.1 Language and Visualization

In the teaching and learning environment, language is pivotal. South Africa's Department of Basic Education (DBE) (2011: 8) defines mathematics as "…a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships." Mathematics is considered a "universal language" that transcends cultural, political, geographical, linguistic and any socially oriented differences. Thus, in any language, whether a mathematics register is formally developed in that language or not, there exist mathematical concepts derived or awaiting to be derived from local people's ways of living. When mathematics concepts resonate with people's knowledge and livelihoods, mathematics becomes meaningful and alive to those people (Mokgwathi & Webb, 2013). We argue that any lack of connection-making between school mathematics and the day-to-day lives of its recipients detaches mathematics from reality, thus making it an abstract and detached subject that is mainly learnt for its own sake. One way to make connections between mathematical ideas and the real world is to visualize these ideas in a real life context.

Teachers generally make use of words to awaken curiosity and stimulate the imagination in the minds of learners. These imaginings often translate to specific images or visualizations as the learners' attempt to make concepts visible (Boaler, Chen, Williams & Cordero, 2017). It follows that the mental images that learners create, then form the basis of how they visualize mathematical concepts and ideas. For the purpose of this paper, emphasis is placed on visualization as an act or process of putting or representing mathematical concepts in a visible form internally in the mind, externally on paper or through other technological tools. Mathematics teachers, especially in multilingual classes, need to be sensitive to the idea that their use of verbal language helps learners generate their own appropriate images of mathematical concepts. In speech, words often cause listeners to retrieve specific visualizations of targeted objects. In this regard, Mesarosova and Mesaros (2011) assert that a verbally evoked image is a fictive image or representation of a mathematical concept that is induced by the teacher in the learner´s conscience mostly by means of verbal impulses.

Naturally, such teacher language practices take advantage of learners´ prior knowledge or experience of the concept taken from their day-to-day lives and contexts.

While some researchers in mathematics education argue that not every learner is a visualizer (Presmeg, 1986, 2006), cognitive psychology posits that everyone uses visual pathways when working on mathematical tasks (Boaler et al., 2017). Thus, teaching should take advantage of these visual pathways in the minds of learners when helping them to understand and make abstractions of various mathematical concepts. As Judson (2016) so aptly articulates, to achieve meaningful conceptual teaching and understanding for all, imaginative mathematics teachers identify and stimulate with words those visualizations embedded in mathematical concepts in the curriculum.

Visual images used to create visual language are inherent in mathematical concepts whether they are verbally or pictorially represented. Visual language in mathematics is a process of communicating a mathematical concept through the concurrent use of images and text. It involves a very close, careful integration and symbiosis of words and images. When one is removed, the remaining communication can become unclear. Thus, as argued by Murphy (2009: 5), "visual images underlie the language of mathematics at every turn." Studies have shown that the integration of verbal and visual elements yields better results than when text is presented separately from visual elements (Mayer, 2003). The use of only text or visuals may not achieve the intended results. We thus argue that when teachers use illustrations from the pupils' first language (that is from their immediate home context) for mathematical purposes, this authenticates, legitimizes and concretizes learners' learning. The importance of using visual representations in mathematics education needs to be seen against the contribution it makes to the development of understanding and intuitional perspectives (Mesarosova & Mesaros, 2011). Using or evoking correct visuals in learners' minds allows them to see and understand mathematical relationships. Teachers' use of appropriate language that draws on pupils' familiar images enables connection making.

Sadoski and Paivio (2013) agree that when concepts are verbally stated, they come with visualizations or representations attached that help explain, interpret, or clarify them. Ideally, such visualizations, when awakened do not conflict with the words or concepts — they are instead an inherent part of the concept. When images or visualizations, numbers and words concurrently participate in the same system of communication, effective teaching of mathematics can take place. Murphy (2009) argues that the use of all three parts of the system — words, numbers and visualizations — results in better comprehension of mathematical concepts during teaching. This model inextricably suggests that visualizations are the embodiment of language. When language and visualization co-emerge, effective learning can take place. We argue that this enactivist notion (Li, Clark & Winchester, 2010) suggests that without language no visualization can take place and vice versa.

Research has shown that language that is intertwined with visual learning and thinking approaches is beneficial to learners who are taught mathematics in a language other than their first language (Judson, 2016; Marentette, 2018). In addition, Marentette (2018) for example asserts that creating visual explanations improves retention of information and deeper understanding of complex concepts. In light of this, students thus benefit from learning through visual forms of communication. When the visual, verbal and the numerical co-emerge, conceptual understanding is achieved. In order for teachers to capitalize on learners' experiences and prior knowledge where teaching takes place in the second language, there are moments when teachers should use learners' first language to help them remember and relate to these experiences and thus make connections to the taught concepts. In such instances, the choice of words and phrases that evoke interesting and relevant images that resonate with a given concept is thus crucial. The presentation of mathematical information verbally, numerically and visually helps children to comprehend complex mathematical information (Murphy, 2009). Murphy (2009) also observes that when teachers express mathematical ideas so that their words, associated images and numbers seamlessly work together, then true communication results.

In a situation where learners are taught in their second language it is particularly important that teachers provide visual cues, graphic representations, gestures, regalia, and pictures during their teaching. It is argued that such practices offer learners the required opportunities to manipulate objects and images to master mathematical concepts and vocabulary.

2.2 Conceptual understanding

Conceptual understanding is one of the key goals of school learning. Kilpatrick, Swafford, & Findel (2001: 120) define conceptual understanding as an integrated and functional grasp of mathematical ideas and argue that "conceptual understanding frequently results in students having less to learn because they can see the deeper similarities between superficially unrelated situations". Using words, connections are made between seemingly unrelated mathematical ideas. According to Kilpatrick et al. (2001), there are four key indicators of conceptual understanding from the both the teachers' and learners' perspectives. First, a teacher who focusses on developing conceptual understanding in learners has the ability to build on learners' prior knowledge. This prior knowledge includes what learners learnt in previous lessons as well as what they can visually recall and use from their home environments. Second, such a teacher is able to help learners make connections because "the degree of students' conceptual understanding is related to the richness and extent of the connections they have made" (Kilpatrick et al., 2001: 119) among different mathematical concepts. The third indicator is the ability to relate mathematical concepts to the real world. Lastly, a teacher who is able to visually and verbally represent mathematical situations in different ways, knowing how different representations can be useful for different purposes, encourages conceptual understanding. We adapted Kilpatrick et al., (2001)'s indicators of conceptual understanding to analyse teachers' language practices in our analysis of data in this study.

3. Theoretical Framework

The challenge for today's mathematics teacher to make learning a meaningful and worthwhile experience for the learner is complex, especially where children of diverse linguistic and socio-cultural backgrounds and cognitive abilities are taught in multilingual classes. As this paper sought to understand teachers' use of learners' first language through code switching in order to stimulate and support visual thinking, the socio-cultural backdrop of the study can thus not be ignored. Consequently, the study was informed by the situated-sociocultural theory as envisaged by Moschkovich (2002), specifically with regard to the role of language in multilingual classroom communication and cognitive development.

The situated perspective in this study describes teaching and learning mathematics as using multiple material, linguistic and social resources (Edu-Buandoh, 2012). This study refers to and suggests that language used for teaching should be "local, grounded in actual practices and experiences" (Gee, 1999: 40). Teaching and learning are inherently social and cultural and participants in the process bring multiple views and representations to the situation, resulting in multiple meanings that are then negotiated by conversations (Moschkovich, 2002). Knowledge in this case is understood to be embedded in and connected to the social and cultural context in which it is developed. Teachers' use of words in learners' first language is situated within specific contexts in a society to convey situated meanings. While embarking on globally accepted mathematics concepts, teaching should happen as a function of its surroundings and the social and cultural experiences of people in which teaching takes place. Collins (1988) refers to situated learning as the notion of learning skills and knowledge in contexts that reflect the way they will be used in real life. In South Africa's township and rural schools, code switching is a norm and prevalently practiced in and outside school life. In classes where code switching occurs, meaning is regarded as an ongoing social process enacted by the use of language and is appropriated by teachers and learners for enhancing understanding (Edu-Buandoh, 2012). Moschkovich's combination of Gee's situated meaning and elements of the sociocultural theory is in this paper regarded as a framework that helps to understand teachers' use of learners' first language, to evoke visual thinking in multilingual classes where the majority of learners learn in their second or third language.

Central to Moschkovich's situated-sociocultural theory is the recognition of the pivotal role played by two or more languages in each situation as a resource to communicate mathematically. The theory describes languages used in communities as a resource for teaching, and multilingualism as a competency in mathematical communication. The situated-sociocultural theory views teaching mathematics as a social activity that employs multiple materials, linguistic and social resources to enhance instruction (Moschkovich, 2002). The theory assumes that mathematics teachers' language practices should be grounded in practice while at the same time connected to mathematical concepts.

The theory also views mathematics learning as participation in a community where learners are taught to mathematize situations, to communicate about these situations and to use resources available to them for mathematizing and communicating (Moschkovich, 2002). The major resources learners bring to their learning environment are their home language and everyday life experiences. Again, Moschkovich (2002: 207) asserts that "some of the resources [multilingual] students use to communicate mathematically are gestures, objects, everyday experiences, their first language, code switching, and mathematical representations."

Second, using a situated-sociocultural perspective permitted us to ask what teacher mathematical language practices were relevant to visual teaching and learning of mathematics, trigonometry and geometry in this case. The situated-sociocultural perspective was suitable because first, it made it possible for us to consider the teachers' use of non-language resources, which includes visuals drawn from the learners' experiences and environment.

Third, this theory enabled us to carefully consider the situational context of teacher language. In mathematics, some of the words and phrases have multiple meanings that may only be differentiated by considering context. During teaching, the use of such words and phrases occur within given social contexts and as explained by Moschkovich (2002: 195), "… much of the meaning of an utterance [teacher language] is derived from the situation." Thus, in considering teacher language in this study, we considered how resources from the teachers' different contextual situations were used to evoke visual thinking and promote meaning making.

Fourthly, a situated-sociocultural perspective helped in opening the way to see language practice as a competence. Analyzing teachers' language use from a situated-sociocultural perspective enabled us to view the teachers' language use as a resource that connected it to the learners' environment, to communicate mathematics during teaching. The teachers' connection-making of everyday scenarios to mathematical language practices helped the study to widen what counts as competence (Moschkovich, 2002). Teachers' reference to and use of everyday scenarios in learners' first language to evoke visual thinking for mathematical purposes was considered a competence. As Moschkovich (2002) suggested, a situated-sociocultural perspective enabled us to avoid a deficiency model of multilingualism, developing detailed descriptions of the linguistic and visual resources that teachers used to communicate mathematically.

4. Sample and Research Process

This study sought to gain a detailed view of three selected teachers' verbal and visual language practices. Their teaching of multilingual mathematics Grade 11 classes was analysed for conceptual understanding. Three Grade 11 mathematics teachers from three districts in the Eastern Cape Province of South Africa participated in this study. The study followed a case study design with each teacher and his/her class constituting a case. The three teachers identified as Teacher A, Teacher B and Teacher C were purposively selected. We specifically identified teachers who were willing to participate, were fluent in isiXhosa, taught at schools where code switching was prevalent and had a rich code-switching practice. We also identified teachers with at least five years' experience of teaching mathematics at secondary level, and were therefore well experienced. This was to rule out the possibility that their language practices might be due to a lack of teaching experience or recognized qualifications. This we did by visiting schools and talking to teachers for three weeks prior to the commencement of data collection process. We also relied on the first author's experience whose work involved interacting with schools in the Eastern Cape and their teachers.

Data were obtained through observing five consecutive geometry (parallel lines and triangles, circle geometry and cyclic quadrilaterals) or trigonometry (sine, cosine and area rule, trigonometric graphs) lessons per teacher. Lesson observations were used to identify language practices of these teachers. At the end of each lesson, each teacher was interviewed. The interviews focused on language practices that the participating teachers demonstrated during the lesson. Lesson observations were video recorded while interviews were audio recorded.

Having sought ethical clearing, together with the consent and approval by the Department of Basic Education, school principals and the teachers, five lessons of each teacher were video recorded. A professional transcriber who is an isiXhosa first language speaker transcribed all the lesson observation videos and the interview audio recordings.

5. Data Analysis

Each recorded video was analyzed qualitatively using the four conceptual understanding indicators adapted from Kilpatrick et al., (2001). First, we looked at how teachers were building on learners' prior knowledge. That is, when teachers, through code switching, used verbal language and visualizations that were already familiar to learners. Secondly, we looked for evidence of teachers' use of mathematical connections within the subject. We looked at the teachers' use of verbal language and visualizations to establish connections between different mathematical concepts and relationships. The third indicator looked at mathematical connections to the real world. Our focus was on the teachers use of everyday isiXhosa verbal language and visualizations to connect mathematics to the real world when explaining mathematical concepts. Lastly, we looked for the use of multiple representations where teachers used verbal language to represent mathematical concepts in multiple visual ways.

During the lesson observations, we focused on isiXhosa words and phrases that the teachers used for each of these four indicators. We then identified those words and phrases that were used to evoke visualizations to enhance conceptual understanding during teaching. For example, Teacher C said "*Jonga for ucalane okwisangqa, all corners should touch i-circumference."* (Look for the cyclic quadrilateral, all corners should touch the circumference). Here we considered use of *ucalane okwisangqa* as drawing on prior knowledge of the learners. They already know *ucalane* (four-sided) and *isangqa* (circle). Teacher C combined those two familiar isiXhosa terms to describe the cyclic quadrilateral. In this study we considered such teacher language use as promoting conceptual understanding.

We also focused on verbal scenarios, descriptions and illustrations that teachers took from their learners' local environment such as the railway line and fencing poles, and how they used them to promote conceptual understanding during their teaching. A situated-sociocultural perspective provided us with lenses to look at words and phrases teachers used, situations/contexts in which the language was used, situated meanings of teacher language and non-language resources that teachers used during teaching. Trends and patterns of words, phrases, descriptions and illustrations that emerged during the lesson observations were then followed up during the interviews. The three participating teachers were interviewed five times each, that is, after every lesson observation. The situated-sociocultural perspective thus allowed us to see and understand some of the complexities and competences (Moschkovich, 2002) exhibited by the participating teachers' language use to enhance visual thinking.

6. Validity and Ethics

Validity was ensured by using multiple sources of evidence such as lesson observations and interviews. In order to ensure the integrity of the language used in the lessons, transcriptions were done by an experienced transcriber and were verified by two university academics who are isiXhosa first language speakers. All ethical protocols as prescribed by the ethics committee of our university were adhered to.

7. Findings and Discussion

In this section, the findings are presented under the four indicators for conceptual understanding discussed above. These are: building on prior knowledge, connection making within mathematics, connecting mathematics to the real world and use of multiple representations.

7.1 Building on prior knowledge

In this section, we focused on some of the everyday words the teachers used to promote conceptual understanding of concepts through building on learners' prior knowledge. The words we focused on were key and relevant for solving given tasks. We focused on the words *fumana* and *ayifumaneki*, as examples of words that were used to enhance visualization.

7.1.1. Fumana - Find

We observed that all three teachers extensively used *fumana* in this study to ask questions. *Fumana* is used in everyday life when someone is asked to find something, whether it is hidden or can be easily found. In mathematics, *fumana* (find) means using mathematical methods to obtain, locate, solve, detect or gather the values, quantities or any mathematical construct in question. Thus, in situations that needed learners to calculate or simply to observe and state required answers, *fumana* was used. Table 1 shows extracts from their lessons:

Table 1: Excerpt 1

Teacher	Actual Teacher Language	English Translation
A	Ku-ABD, sizamfumana njani u-BD?	For ABD, how can we find BD?
A	Now calculate BC. Sizamfumana njani u-BC?	Now calculate BC. How are we going to calculate BC?
B	Cofa ecalculatini yakho, kwi-reference sifumana bani?	Use your calculator, what do we get as a reference?
B	Sizawuyifumana njani i-value ka-A there?	How do we find the value of A there?
C	Xa u multiplier u “b” no “a”, surely uza fumana u “ba”	When you multiply “b” and “a”, surely you will get “ba”

The everyday use of *fumana* retains the same meaning as when used for mathematical purposes. Thus teachers capitalized on learners’ prior knowledge and understanding of this consistent and precise meaning of *fumana*. In this study, all observed teachers used *fumana* frequently to mean ‘calculate’, ‘find’ or ‘what we get.’ Hence *fumana* has multiple meanings both in everyday use and when used for mathematical purposes. The ability to use everyday words in a mathematical way has advantages of visually bringing learners’ prior familiar mental pictures into the formal teaching context of the classroom. Because *fumana* carries the same meaning in both everyday life and the mathematical realm, it helped the teacher to make mathematical connections. During the interview, Teacher C said “*everyday terms that have the same meaning when used to teach maths helps link learners’ life outside school with mathematics*.” Home language terms with multiple meanings are also important because they can be used in various contexts for conceptual understanding during teaching. Moschkovich (2002: 194) asserts “because there are multiple meanings for the same term, students who are learning mathematics can be described as learning to use these multiple meanings appropriately.” Viewed from such a perspective, the use of these familiar isiXhosa terms offered more advantages than limitations. In the South African education system, *fumana* is also used throughout school from the foundation phase onwards – hence the advancement of connections within and across school grades. Considering the social context in which *fumana* was used, learners were appropriately prompted to seek solutions to given tasks.

7.1.2 *Ayifumaneki*- Undefined

Another mathematical concept we observed that utilized learners’ prior knowledge and experiences was that of an ‘undefined’ situation. Teachers used it when they were teaching and during the interviews. Teacher B explained that ‘undefined’ was translated to in isiXhosa (Table 2). The term ‘undefined’ was code switched as *ayifumaneki* by Teacher B, meaning ‘that which we cannot find or get.’

Table 2: Excerpt 2

	Actual Language Spoken	English Translation
Researcher	You solved and the gradient was undefined, what is the IsiXhosa term for 'undefined?'	
Teacher B	Undefined ayifumaneki, you can't get to the solution. Ayifumaneki, you cannot define it. Ja it doesn't exist, ayikho, ayifumaneki. (Interview 2)	Undefined, you cannot get it, you cannot get the solution. The thing is you cannot define it. Yes it does not exist.

The use of *ayifumaneki* helped to enlighten the concept of an undefined situation. Additionally, *ayifumaneki* was intended to help learners visualize that some situations, like the gradient of a vertical line, are undefined. This can also be applied to other cases in mathematics that are undefined or where a solution cannot be calculated or found. The use of familiar everyday words in the learners' home language for mathematical purposes may promote conceptual understanding as learners are prompted and assisted to visualize situations that may not be otherwise easy to understand. Teachers chose words that were from their daily use, familiar to learners and relating to their life outside school.

7.2 Connection-making within mathematics

This section focuses on one example of an isiXhosa word that the participating teachers used to make connections within mathematics. This is the word zoba/krwela- draw/sketch.

7.2.1. *Zoba/Krwela* - Draw/Sketch

One of the terms that was used during our observations was the word *zoba* (Table 3), the translation for 'draw' or 'sketch'. In everyday life, *zoba* is used when referring specifically to drawing a sketch or a diagram. The term *zoba* has no other meaning, unlike the English translation 'draw', which has various meanings depending on the context and thus can be ambiguous. Once *zoba* has been uttered however, learners who are isiXhosa first language speakers immediately feel compelled to produce a picture or diagram because there is no ambiguity. Because *zoba* does not have other meanings or representations it may assist the listener to directly connect it to actual mathematical processes.

Table 3: Excerpt 3

Teacher	Actual Teacher Language	English Translation
B	Makaze omnye azosizobela i-trapezium	One of you must come and draw a trapezium.
A	U-B ngu-40, usezantsi komgca.	B is 40, it is below the line.
C	So one full Tan graph nantsi i-shape yayo, xa uzizobela yona yijonge.	Here is the shape, when you draw it.
A	From B uye ku-E krwela umgca, a straight line, engekho goso.	Draw a straight line from B to E that is not crooked.

Teacher A used the phrase *krwela umgca* to refer specifically to the drawing of a straight line and not any other form of drawing (see Table 3). This phrase requires one to draw only a straight line. The words *umgca* and *komgca* visually suggest and encapsulate a 'line'. Teacher A in the fourth extract in Table 3 required learners to draw a straight line and she ended by saying '*engekho goso'* meaning 'that is not bent or crooked.' That last part explicitly directs learners to envision and draw a straight line (and not a crooked line). This was done to connect and emphasize the mathematical idea of a straight line. Presented here are examples of precise terms in isiXhosa that connect mathematical concepts. We argue that their use through relating to real life and learners' prior knowledge facilitates conceptual understanding.

In this study, we contend that the use of such isiXhosa terms helps learners to visualize the concepts and make connections because of the immediacy and familiarity of the terms to isiXhosa first language speakers. This in turn is considered in this study as having a higher prospect of promoting conceptual understanding.

While teachers used some precise isiXhosa terms that enhanced visualization opportunities, such terms were not used extensively in the lessons during the teaching of geometry and trigonometry. We observed that teachers used everyday terms precisely for mathematical purposes. Technical terms such as 'perpendicular', 'parallel', 'intersection', 'colinear', 'period', 'asymptote' and 'quadrant' were all borrowed. We posit that the use of borrowing still presents terms that may not be familiar to learners as they are not in a familiar language — thus creating fewer opportunities for connection-making. During interviews, Teachers B said "*Some terms needs one to think carefully before translating like 'asymptote' that I used in my lesson. I never translate it to umgca owe-function osondela kufuphi kodwa ungade uthinte ndawo (a line that the function keeps getting close to but never actually touches). But this translation makes more sense."* When the majority of learners are being taught in the LOLT that they are less proficient in, the use of familiar isiXhosa terms through code switching stimulating visual elements of a concept, should be especially encouraged. This should be done for both everyday terms used for mathematical purposes as well as geometrically and trigonometrically specialized terms.

7.3 Multiple representations

Observed teachers reiterated the paucity of isiXhosa terms for some mathematical concepts in the teaching of geometry and trigonometry (Table 4). They concurred that some mathematical terms were not easy to translate into their home language. In such situations where the mathematical concept lacked an equivalent word or phrase in isiXhosa, teachers said they employ either borrowing, descriptions of those words, representations or illustrations from the learners' environment. Such language practices where teachers used multiple representations were perceived as efforts to help learners visualize and understand mathematical concepts.

Table 4: Excerpt 4

	Actual Language Spoken	**English Translation**
Researcher	What language related challenges did you face during the teaching of geometry?	
Teacher A	Most of *ama-concepts* that I used, were in English, I could not find them in Xhosa. I don't know *i-tangent* in Xhosa *lo-perpendicular*. There is no one word for some of these words in Xhosa unless I describe or illustrate them.	Most of the concepts that I used, were in English, I could not find them in Xhosa. I don't know a tangent in Xhosa or perpendicular. There is no one word for some of these words in Xhosa unless I describe or illustrate them.
Teacher B	*Amanye amagama ku-geometr*y are hard to find Xhosa equivalency. In geometry I just borrow if I can't find a word in Xhosa.	Some of the geometry concepts are hard to find Xhosa equivalency. In geometry I just borrow if I can't find a word in Xhosa.
Teacher C	Some words *anzima* to translate. Like center, *Phakathi* is used but means in the middle. *i-Obtuse* or *i-reflex* angles are difficult to say in Xhosa thus I just borrow when I teach.	Some words are not easy to translate. Like center, *Phakathi* is used but means in the middle. Words like Obtuse or reflex angles are difficult to say in Xhosa thus I just borrow when I teach.

Table 5: Excerpt 5

	Actual Language Spoken	**English Translation**
Researcher	How do you explain the concept of parallel lines in class using their home language?	
Teacher B	Imigca enxuseneyo are parallel lines. I normally relate parallel lines with iziporo zikaloliwe. If you look at iziporo pha ku-train, they will never meet even if they are taking a turn they all taking a turn. So basically it's another visual and familiar example of parallel lines. Even umgwaqo lapha ku-freeway, your lines that are there on the freeway are also parallel (Interview 2).	Lines that are close to each other are parallel. I normally relate parallel lines to railway tracks. If you look at the railway lines, they will never meet even if they are turning, they all turn. So basically it's another visual and familiar example of parallel lines. Even the roads, the freeway, your lines that are there on the freeway are also parallel.
Teacher C	Look at the train, pha i-train mos ineziporo zibini (has two rail lines), if those two would try to meet, it would get dangerous and people would die. So those two tracks are parallel. This will give them confidence as you use known visible things around them. (Interview 2).	Look at the train's tracks, has two rail lines, if those two would try to meet, it would get dangerous and people would die. So those two tracks are parallel. This will give them confidence as you use known visible things around them.

A trend that emerged with all participating teachers was their use of code switching for illustrating a point by using everyday scenarios (see Tables 4 and 5). In the interviews, we followed up on this tendency with Teachers B and C when they were dealing with parallel lines. Two everyday examples to illustrate parallel lines were given – railway lines and lines on a freeway (Table 5).

During interviews teachers reiterated the need to tap into locally available everyday visual illustrations that are able to facilitate visualizations of mathematical concepts. Learners in these classes were familiar with railway lines and highways, multiple representations teachers used for parallel lines. The teachers thus suggested that these would be appropriate phrases to use to stimulate the visualization of the parallel lines concept. Both teachers agreed that any scenario where two lines do not meet would be suitable to illustrate parallel lines. The use of such illustrations that shows a multiplicity of representations from everyday life assists learners to internally visualize embedded mathematics concepts.

In Tables 4 and 6, Teachers A and C respectively concurred that certain concepts like 'perpendicular' were not easy to explain in the home language, except if the teacher used indigenously developed visual illustrations such as games, songs in isiXhosa or some day-to-day experiences familiar to learners, to help them understand the concepts. They argued that the indigenous equivalent translations of some of these mathematical terms are not readily available, and if they are, the isiXhosa translation is not familiar to learners. Teacher C cited a song that learners sing outside school that may be used to make the concept of 'perpendicular' more understandable and real to learners. Teacher C agreed that picking scenarios from everyday life would help learners to understand better.

Table 6: Excerpt 6

	Actual Language Spoken	**English Translation**
Researcher	How do you explain the concept 'perpendicular' to your learners?	
Teacher C	I relate to an event though not necessarily perfect to represent perpendicular. During protests there is a song that says 'up the gear 90^0'. People dance showing something of 90^0 with their legs. You can draw an event, an old event to just demonstrate. (Interview 3)	I relate to an event though not necessarily perfect to represent perpendicular. During protests there is a song that says 'up the gear 90 degrees'. People dance showing something of 90 degrees with their legs. You can draw an event, an old event to just demonstrate. .
Teacher A	I can take them to the netball or soccer field, nhe, so that they can observe ama-features of i-perpendicular. I can also use the poles, goal posts. At each corner it's straight, there is 90^0 you know, so there is perpendicular there. (Interview 3)	I can take them to the netball or soccer field, right, so that they can observe features of a perpendicular situation. I can also use the poles, goal posts. At each corner it's straight, there is 900 you know, so there is perpendicular there.

Teacher C indicated that find a perfect translation for 'perpendicular' in isiXhosa that would visually represent the concept. Teacher B explained that "*imigca ethe nkqo iya nqamlezana at right angles (straight lines intersecting at right-angles)*" describes perpendicular lines.

Thus Teachers A and C agreed that various illustrations stimulating visual representations would make the concept clearer and assist learners, especially if these were taken from their immediate environment. This emphasis on inherently socially negotiated examples and illustrations aids conceptual learning, thinking and knowing, as learners engage in activities arising from the socially and culturally immediate world (Lave & Wanger, 1991).

The lack of precise mathematical terms and the lack of a mathematics register in isiXhosa encouraged and forced the teachers to use illustrations that would evoke visual impressions of concepts in the learners. These illustrations were taken from learners' everyday environments familiar to learners. The teachers used mostly isiXhosa when giving these illustrations. For this study, it was important to understand how mathematics teachers' everyday illustrations were used to provide linguistic resources for mathematical communication and conceptual understanding.

8. Summary and Conclusion

Participating teachers in this study used everyday vocabulary to promote visualization of mathematical concepts in the learners' first language to stimulate visual thinking, for conceptual understanding of trigonometry and geometric concepts. Learners' first language was used as a way of building on prior knowledge — it was used to make connections within mathematics and with the real world. Teachers also used terms that promoted multiple representations of mathematical concepts. It was found in this study that while teachers did not make extensive use of the learners' first language to help promote visualizations, they used everyday words which were familiar to their learners, thereby building on their prior knowledge. Use of such terms is said to result in effective learning because the learners can see the connections between situations that may seem unrelated, resulting in rich conceptual understanding (Kilpatrick et al., 2001). We argue that teachers' use of borrowed words, especially of technical trigonometric and geometric terms, presented fewer opportunities for their learners to make visualizations, build on their prior knowledge and to make connections.

Interviews in this study revealed that the observed teachers did not plan on how and when to use the learners' first language – it occurred spontaneously. We contend though, that the use of language to promote visualizations of mathematical ideas during teaching should be strategic, well planned and prepared. What this observation means is that when teachers are planning lessons for English second language learners of mathematics, careful selection of words and phrases that are familiar to learners, used in their everyday lives and have the potential to evoke appropriate visualizations of concepts, should be carefully considered. Teachers of mathematics who understand how words can be used to visually stimulate conceptual teaching, should be conscious of choosing words that utilise imagery to enhance learning. The upshot of all this is illuminated by Horn (2001: 1) who posits that "when words and visual elements are closely entwined, we create something new and we augment our communal intelligence... visual language has the potential for increasing 'human bandwidth' — the capacity to take in, comprehend, and more efficiently synthesize large amounts of new information."

Teachers also used descriptions of mathematical concepts in their home language in those situations where they could not find an immediate suitable isiXhosa word for some geometric and trigonometric concepts. These illustrations were mainly from learners' prior knowledge. Teachers should utilize visual resources existing in their locality, especially those familiar to their learners, to develop conceptual understanding, enable these learners to grasp underlying concepts and learn more effectively. Teachers of mathematics should carefully choose their language and visualizations to demonstrate precisely intended mathematical ideas if they cannot bring the actual artefacts into class.

The teachers' use of visualizations in their teaching enhancing conceptual understanding of geometry and trigonometry concepts is commendable and should be encouraged. In multilingual mathematics classes, making connections between different representations is critical in developing conceptual understanding (Smith, Bill & Raith, 2018). However, these should be well planned in order for such practices to yield the best results. Ad hoc illustrations may be more harmful if not properly planned and well thought out. A well-reasoned, sentient and astutely designed visual illustration mixed with verbal descriptions can yield a much more powerful and memorable learning experience than only verbal or textual descriptions.

While the use of everyday words, phrases and visualizations has many socio-cultural advantages, teachers should ensure that mathematization is prioritized and achieved through such efforts, especially at higher grades of school learning. Pre-service and in-service teachers should be taught to use language strategically so that learners of mathematics will not see it as a 'foreign language' that has nothing to do with the day-to-day lives of those learning it. This is possible when teachers deliberately invest time and effort in finding appropriate and suitable words as well as illustrations taken from everyday life that clearly and vividly represent the relevant mathematical concepts.

As may be the case in other learning environments in South Africa and elsewhere, teachers of the Grade 11 classes used in this study tended to avoid bringing actual concrete visual aids during teaching and favoured verbal and abstract approaches. Thus, the ability to use verbal language that evokes visual images appropriate to a given task is necessary. We argue that there is thus a need for providing adequate and appropriate academic language support to teachers of all learners learning mathematics in a language that is not their first. Teachers would need to be trained to harness linguistic resources that evoke mathematically related images from their environment for the improvement of mathematical teaching. Such support mechanisms should include topic-specific language demands, since each mathematics domain has its own sub-register. Until a mathematics register in isiXhosa is developed and agreed upon, there is also a need to explore ways to support teachers of second language learners how to better engage and address the needs of multilingual learners in the mathematics classroom.

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